

# Selecting an Appropriate Weighting Strategy in Maximum-Accuracy Time-to-Event (Survival) Analysis

Paul R. Yarnold, Ph.D., Nathaniel J. Rhodes, Pharm.D., M.Sc.,

Optimal Data Analysis, LLC

Chicago College of Pharmacy, and the

Pharmacometrics Center of Excellence, Midwestern University

and

Ariel Linden, Dr.P.H.

Linden Consulting Group, LLC

Different weighting schemes in optimal survival analysis are considered.

Consider an optimal<sup>1,2</sup> survival analysis<sup>3,4</sup> for a sample of subjects observed for a finite number of consecutive time periods: 1, 2, 3 ... X.

If observations are weighted by  $X_i$  then the non-censored surviving observations which avoided the outcome for X periods are weighted more heavily than observations that achieved the outcome in fewer than X periods, all other things being equal (e.g., this does *not* assume a complex weight, such as the value  $X_i$  multiplied by a propensity score<sup>5</sup>). That is, in this scheme observations that *did NOT achieve* the outcome are weighted most heavily. For example, if the outcome is death, then observations that lived the entire X periods are weighted more heavily than observations that perished in fewer than X periods. This weighting scheme favors accurate classification of *surviving* observations.

In contrast, if observations are weighted by  $1/X_i$  then non-censored observations that did

not survive (i.e., achieved the outcome in fewer than X periods) are weighted more heavily than observations that failed to achieve the outcome in X periods, all other things being equal. That is, in this scheme observations that *did achieve* the outcome are weighted the most heavily. For example, if the outcome is death, then observations that perished in fewer than X periods are weighted more heavily than observations that perished (or failed to perish) in X periods. This weighting scheme favors accurate classification of *non-surviving* observations.

Finally, consider analysis that observes a sample of subjects over a fixed time period of *exactly* X time periods, which does *not* weight observations by time-to-event, but rather instead weights all observations equally by unity (or by prior odds<sup>1,2</sup>, by a propensity score<sup>6,7</sup>, and/or by a measure of value<sup>8-11</sup>). The objective of such an analysis is to contrast observations who live vs.

perish over  $X$  or fewer units of time. This weighting scheme does not favor the accurate classification of *surviving vs. non-surviving* observations, but instead provides a means of ensuring that the treatment groups are comparable.<sup>12-14</sup>

Most classification models seek balanced sensitivity and specificity so as to maximize ROC area.<sup>15-20</sup> As always, the crucial question is—is the model fit for purpose? Thus, the decision to weight outcomes more heavily *vs. non-outcomes*, to do the opposite, or to weight observations equally is entirely a question of whether the goal is to predict the outcomes or the non-outcomes most accurately. A related caveat is the problem familiar to data scientists—the rare event case caused by base rates of  $<10\%$  of the sample: in this scenario the event rate is low and often the “best guess” for an empirical model is to guess “non-event”.<sup>21</sup> There are many alternative ways that this problem may be addressed—ranging from complex simulation based approaches (e.g., SMOTE) to the random under- and over-sampling of training datasets.<sup>22</sup> However, we believe that simpler weighting schemes (e.g., at risk time, propensity, value) are highly intuitive and thus offer translational clarity for end-users. That is, rather than presenting “black box” solutions for class imbalance, the model offers insight into the factors that end-users should consider as being more influential over the final outcome classification. Finally, it is important to consider the “bias” of predictions: is a model with significant bias acceptable for use? To answer this, an end-user must consider the consequences of false positives and false negatives.<sup>23</sup> The impact of false signals is clearly relevant to the clinical or use case scenario. For example, consequences of a false positive prediction for invasive breast cancers based on mammogram, antibiotic resistance in pneumonia based on prior hospitalization history, bridge stability according to electrical conductance from wire supports, or hostile inbound aircraft using radar signals are

all very different from a false positive prediction for a tooth cavity based on dental X-ray imaging data.

Weighting strategies discussed presently do different things, for different purposes. The propensity score and similar types of weights, for example prior odds, are used to ensure that treatment groups (or exposure groups in epidemiology) are comparable and are therefore exchangeable. Weighting by value is used to ensure that differences between observations are accounted for in the hopes of maximizing the omnibus positive gain (return) or minimizing the omnibus negative gain (loss) for a group. And, weighting by time is used to understand the etiology of an outcome: for example, in some cases the longer subjects are monitored the more likely they are to contract an illness or die, whereas in other cases subjects may be more likely to contract the illness or die relatively quickly—and thus the longer that they are monitored, the more likely it is that they have passed through the period of risk. These types of adjustments may be made using parametric survival models whereby a researcher chooses the distribution model believed to best represent the trajectory of acquiring the outcome. In contrast, maximum-accuracy methods require no distributional assumptions, and instead simply identify the model(s) which explicitly maximize model accuracy in predicting the outcome being investigated.

An empirical demonstration of these alternative weighting schemes is welcomed.

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### Author Notes

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