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Confirming the Efficacy of Weighting in Optimal Markov Analysis: Modeling Serial Symptom Ratings

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This paper uses ODA to weight each event in the transition table by its corresponding absolute change-in-value, thereby maximizing precision of the class variable as well as model accuracy.

Used to model an ordinal outcome a Markov Model may be specified as the least granular ordered configuration.[1] For example, to model serial symptom ratings made by a single person, for each trial the rating is indicated as being lower (less severe symptom rating at time *i*+1 *vs*. time *i*), or unchanged/up (unchanged or more severe symptom rating at time *i*+1 *vs*. time *i*). This is illustrated in Figure 1: "Down" (D) indicates the symptom rating was less severe, and "Not Down" (ND) indicates the symptom rating did NOT become less severe (i.e., was either unchanged or more severe).

Figure 1: Least-Granular Ordered Markov Configuration for Symptom Rating

	Time *i*+1	
Time *i*	Down	Not Down
Down	D-D	D-ND
Not Down	ND-D	ND-ND

Two possible ordered model configurations are *weighted* and *unweighted*. Unweighted designs contrast relative change in symptom rating over successive days—lower rating *vs*. not lower. Weighted designs weight each entry in the Markov table by the absolute value of the change in symptom rating: entries with a zero weight are thus omitted from weighted analysis.[1] Both configurations are illustrated herein for an application involving serial ratings of quality of sleep and amount of pain, made by a patient with fibromyalgia.

Data

Data for this study were obtained by the Self-Monitoring And Review Tool (SMART), an interactive, internet-based, self-monitoring and feedback system which helps individual users discover and monitor links between their own health-related behaviors, management strategies, and symptom levels over time.[2,3] SMART involves longitudinal collection and optimal statistical analysis of an individual's self-monitoring data, and delivery of personalized feedback derived from the data. A total of 46 daily pain and sleep status ratings made by a single individual using a 10-point Likert-type scale (1=not at all bothersome; 10=extremely bothersome) over a 64-day period were used in

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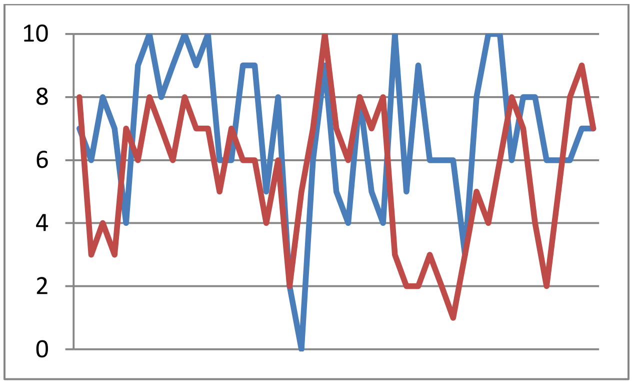
analyses reported here. A plot of the symptoms over time is presented in Figure 2.

Figure 2: Patient's Pain (Red) and Sleep Difficulty (Blue) Ratings by Time



Pain Ratings: Unweighted Analysis

The pain series was used to construct a unit-weighted Markov transition table (Table 1). Data were assessed via non-directional ODA testing the exploratory hypothesis that data fall into one or the other diagonal. The ODA model indicated stationarity (a lower symptom rating at t_i predicts a lower symptom rating at t_{i+1}, and *vice versa*). The model yielded 68.18% sensitivity when classifying Down days, and 72.73% when classifying Not Down days: this result was statistically significant (p<0.016), indicating a moderate level of predictive accuracy (ESS=40.91; 0=chance; 100=no errors).

Table 1: Transition Table for Unweighted Change in Pain Rating

	Time *i*+1	
Time *i*	Down	Not Down
Down	15	7
Not Down	6	16

Pain Ratings: Weighted Analysis

Next, every event in the transition table was weighted by the absolute value of the difference in symptom rating between the index (t_i+1) *vs.* prior (t_i) day: weights must be positive, so the sample was reduced by the number of events having a difference score of 0. The resulting ODA model indicated stationarity, yielding the identical transition table (Table 2) except that the ND-D tally fell to 5, and the ND-ND tally fell to 14, due to elimination of events with a weight of 0. For the weighted model, weighted sensitivity for Down (71.05) and for Not Down (76.92) exceeded unit-weighted findings: the resulting weighted ESS=48.17 is moderate (near the minimum criterion of ESS=50 indicating a relatively strong effect), and statistically significant (p<0.0056).

Sleep Ratings: Unweighted Analysis

The sleep series was used to construct a unit-weighted Markov transition table (Table 2). Non-directional ODA testing the exploratory hypothesis that data fall into one or the other diagonal yielded a model indicating stationarity. The model yielded 76.47% sensitivity when classifying Down days, and 51.85% when classifying Not Down days: this result was not statistically significant (p<0.116), indicating moderate predictive accuracy (ESS=28.32).

Table 2: Transition Table for Unweighted Change in Pain Rating

	Time *i*+1	
Time *i*	Down	Not Down
Down	13	4
Not Down	13	14

Sleep Ratings: Weighted Analysis

Next, every event in the transition table was weighted by the absolute value of the difference in symptom rating between the index (t_i+1) *vs.* prior (t_i) day. The resulting ODA model indicated stationarity, yielding the identical transition table (Table 2) except that the ND-D tally fell to 7, and the ND-ND tally fell to 11, due to elimination of events with a weight of 0. For the weighted model, weighted sensitivity for Down (85.71) and Not Down (76.47) exceeded unit-weighted findings: the resulting weighted ESS=62.18 indicates a relatively strong effect which is statistically significant (p<0.00092).

Comments

These findings clearly show that Markov transition tables are sensitive to measurement precision. Not done presently so as to maximize clarity of exposition, future research in this area should ipsatively standardize data prior to constructing a weighted transition table: this will require arithmetic manipulation to ensure that all weights exceed zero, and then later reversing the manipulation to obtain the desired metric of change. This transformation maximizes ESS and inhibits paradoxical confounding in longitudinal single-subject designs.[4-7] Furthermore, assuming that a sufficient number of events with a weight of 0 exist, the next-greater granularity Markov configuration, adding a category with symptom ratings at times i and i+1 being unchanged (U), should be considered (Figure 3).

Figure 3: Second-Least-Granular Ordered Markov Configuration for Symptom Rating

	Time *i*+1		
Time *i*	Down	Unchanged	Not Down
Down	D-D	D-U	D-ND
Unchanged	U-D	U-U	U-ND
Not Down	ND-D	ND-U	ND-ND

References

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Author Notes

No conflict of interest was reported.