

Novometric Analysis with Ordered Class Variables: The Optimal Alternative to Linear Regression Analysis

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Employed to model an ordered dependent (class) variable, Pearson correlation¹ (r) is used in univariable applications featuring one ordered independent variable (attribute), and multiple regression analysis (MRA) is utilized in multivariable applications featuring two or more attributes. Prior research demonstrated how to maximize the predictive accuracy of univariable^{2,3} and multivariable³⁻⁶ regression models vis-à-vis an ODA-based procedure. The present paper instead demonstrates optimal alternatives to r and MRA.

A variation of the partitioning algorithm (PA) used herein is employed in the analysis of both interrupted and continuous time-series.⁷⁻¹⁰ In a *time-ordered* series PA involves creating a set of dummy codes indicating if each observation (i.e., each data point in the series) was recorded at or prior to (0), or after (1), every sequential step in the time-series.⁷ Similarly, in a *value-ordered* series PA involves creating a set of dummy codes indicating if each observation (i.e., each data point in the sample) had a score equal to or less than (0), or greater than (1), each sequential value in the class variable measurement scale. In either situation each dummy class variable is analyzed with the attribute in a separate novometric analysis, and the model with the largest ESS statistic (if competing models have an identical number of strata), or the smallest D statistic (a normed index of distance separating ideal versus empirical model in the application),

is selected as the globally optimal (GO) model of the relationship between the class variable and the attribute for the sample.^{2,11}

Pearson r versus Novometric Analysis

Exposition begins with two examples of the maximum-accuracy alternative to r .

Age and Patient Comorbidity

Data for this example come from an evaluation of a disease management program designed for patients with congestive heart failure and implemented in a large health plan located in the Western United States.¹² The primary goal of the intervention was to reduce avoidable hospitalizations.¹³ Two variables from that study are used in the current example; age and the Charlson Comorbidity Index score (CCI).¹⁴ Descriptive statistics for age (14-103)

and CCI (0-17), both measured on integer scales, are presented in Table 1.

Table 1: Descriptive Summary: Age and CCI

Variable	N	Mean	SD	CV (%)
Age	7,971	64.05	15.64	24.41
CCI Index	7,971	2.79	2.56	91.74

Correlation Analysis: The Pearson correlation between age and CCI score is $r = 0.354$, $p < 0.0001$. The coefficient of determination, $R^2 = 0.125$, indicates that 12.5% of the variation in CCI score is explained in terms of this positive linear function. Linear models with an R^2 of this magnitude only accurately predict values of the dependent (class) variable that lie at or near to the sample mean. Such models return an ESS statistic close to zero—the level of accuracy expected by chance. The ESS for this r model is 1.43, indicating an extremely weak effect.^{2,3}

Novometric Analysis: The first axiom of novometric theory requires a sample sufficiently large to provide minimally adequate statistical power to test the *a priori* hypothesis.² Presently, statistical power analysis indicated a minimum of 32 patients in all model endpoints (i.e., model strata) yields 90% power to detect generalized (per-comparison) non-directional $p \leq 0.05$ for a moderate effect.² Models meeting the minimum strata sample size criterion were obtained for dummy-coded CCI scores between 0 and 12 (the minimum and maximum scores used in PA, respectively): thus a total of 13 CCI dummy class variables were constructed and analyzed. Summarized in Table 2, every optimal model that emerged had two strata, each representing more than 32 patients. If ESS declined in cross-generalizability analysis then the LOO estimate is given. All tabled ESS values have $p < 0.0001$.

The model with a CCI cut-point of 2 had greatest accuracy (76.39%) in predicting low (≤ 2) CCI scores, and the model with a cut-point of 12 had greatest accuracy (86.49%) in predicting high (> 12) CCI scores. In LOO analysis the

sensitivity of the latter model was stable for CCI scores ≤ 12 , but fell to 78.38% for higher scores.

Table 2: Optimal Models for 13 CCI-Score-Based Dummy Class Variables

Cut-Points		Sensitivity		ESS	
CCI	Age	Class 0	Class 1	Training	LOO
0	59.5	56.40	63.36	19.76	
1	64.5	74.59	54.99	29.57	
2	67.5	76.39	57.06	33.44	
3	67.5	73.08	64.05	37.14	
4	64.5	64.34	74.61	38.95	
5	64.5	61.99	76.66	38.65	
6	64.5	60.00	77.03	37.03	
7	64.5	58.81	76.89	35.70	26.77
8	68.4	65.22	65.85	31.07	
9	64.5	57.24	74.19	31.43	
10	64.5	57.01	77.59	34.59	
11	64.5	56.82	81.82	38.64	28.04
12	64.5	56.71	86.49	43.19	35.08

The greatest ESS identified in training analysis, 43.19, emerged for a CCI cutpoint of 12, but ESS declined to 35.08 in LOO analysis.

The greatest ESS in LOO analysis, 38.95 (an effect of moderate strength^{2,8}), emerged for a CCI cut-point of 4. This model was selected on the basis of the fourth axiom of novometric theory as being the globally-optimal model in this example.² The model was: if age ≤ 64.5 years then predict CCI score ≤ 4 ; otherwise predict CCI score > 4 . The confusion matrix for this model in LOO analysis is given in Table 3.

Table 3: Confusion Matrix for Predicted CCI Score: Age Cut-Point ≤ 64.5 Years

		Predicted CCI		
		≤ 4	> 4	
Actual	≤ 4	4,097	2,271	64.34%
CCI	> 4	407	1,196	74.61%

As seen, two of three patients 64.5 years of age or younger had a CCI score of four or

less, and three of four patients 64.5 years of age and older had a CCI score of five or greater.

Age and Number of Office Visits

Data for this example come from an evaluation of a health management program that invited individuals with chronic conditions to enroll in a nursing intervention intended to improve clinical indices of care while reducing medical costs.¹⁵ Two variables from that study are used in the current example; age and the number of physician office visits over the study period. Age was reported as number of years, scientifically rounded to two decimals (19.90-87.30). Number of office visits was measured on an integer scale (0-66). Descriptive statistics for these variables are given in Table 4.

Table 4: Descriptive Summary of Variables

Variable	N	Mean	SD	CV (%)
Age	7,868	46.79	11.07	23.67
Office Visits	7,868	3.93	4.52	115.01

Correlation Analysis: The Pearson correlation between age and number of office visits is $r = 0.156$, $p < 0.0001$. The R^2 statistic indicates that 2.4% of the variation in number of office visits is explained as a positive linear function of age. The ESS for this r model is ESS=0.13, reflecting a miniscule effect.

Novometric Analysis: Models meeting the minimum strata sample size criterion were obtained for dummy-coded number-of-visits scores between 0 and 22 (minimum and maximum scores used in PA, respectively): a total of 23 number-of-visits dummy class variables were thus analyzed. Each optimal model summarized in Table 5 had two strata each representing more than 32 patients. If model ESS declined in LOO analysis, then ESS in LOO analysis is given: all of the tabled ESS values have $p < 0.0001$ except for cut-point 12, for which $p < 0.039$ for ESS in LOO analysis. Dashes indicate no statistically reliable model was identified.

Table 5: Optimal Models for 23 Number-of-Visits-Based Dummy Class Variables

<u>Cut-Points</u>		<u>Sensitivity</u>		<u>ESS</u>	
<u>Visits</u>	<u>Age</u>	<u>Class 0</u>	<u>Class 1</u>	<u>Training</u>	<u>LOO</u>
0	44.6	46.26	60.32	6.58	
1	46.2	54.48	55.51	10.00	
2	-	-	-	-	-
3	49.2	64.77	48.65	13.43	
4	49.2	63.72	49.91	13.62	
5	49.0	61.60	51.58	13.19	
6	46.8	53.02	59.93	12.95	
7	-	-	-	-	-
8	46.6	51.42	62.84	14.26	13.77
9	46.8	51.63	64.55	16.18	15.81
10	52.1	70.37	45.92	16.28	14.98
11	58.2	85.74	30.77	16.51	14.38
12	51.6	68.23	48.91	17.14	7.36
13	51.6	68.53	50.34	18.87	17.19
14	51.2	66.76	52.12	18.88	17.18
15	51.6	67.90	52.43	20.34	19.80
16	46.6	49.98	72.41	22.39	
17	-	-	-	-	-
18	52.2	69.65	57.29	26.94	24.86
19	-	-	-	-	-
20	52.2	69.52	57.89	27.41	25.66
21	-	-	-	-	-
22	62.2	92.34	36.11	28.45	25.67

In training analysis the model using a cut-point of 22 had greatest accuracy (92.34%) in predicting the absence of a very high (>22) number of visits: in LOO analysis the sensitivity of this model declined to 33.33% for >22 visits. The model using a cut-point of 16 had greatest accuracy (72.41%) in predicting the presence of a high (>16) number of visits.

Greatest training ESS, 28.45 (a moderate effect), emerged for the number-of-visits cut-point of 22: while ESS fell to 25.67 (a moderate effect) in LOO analysis, it nevertheless was the greatest ESS identified in LOO analysis. This model was thus chosen as the globally-optimal model. The model was: if age ≤ 62.2 years then predict number of visits ≤ 22; otherwise predict visits > 22. Table 6 gives the confusion matrix for this model in LOO analysis.

Table 6: Confusion Matrix for Number of Office Visits: Age Cut-Point ≤ 62.2 Years

		Predicted Visits		
		≤ 22	> 22	
Actual Visits	≤ 22	7,232	600	92.34%
	> 22	24	12	33.33%

As seen, nine of ten patients 62.2 years of age or younger had 22 or fewer office visits, and one of three patients older than 62.2 years had more than 22 office visits.

MRA versus Novometric Analysis

Exposition now turns to an example of the maximum-accuracy alternative to MRA.

Patient Age and Resource Utilization

Data for this example come from the evaluation of a primary care-based medical home pilot program that invited patients to enroll if they had a chronic illness or were predicted to have high costs in the following year. The goal of the pilot was to lower health care costs for program participants by providing intensified primary care that was intended to reduce unnecessary emergency department visits and hospitalizations.^{16,17} Ordered variables were measured on integer scales. Descriptive statistics for ordered and categorical variables are given in Table 7 (OP = other procedures).

MRA Analysis: Age was treated as the dependent measure and modeled as a simple main-effects function of all 12 (i.e., 10 ordered, 2 categorical-binary) independent variables.¹⁸ The MRA model (coefficients are reported to two significant digits to the right of the decimal, when that is possible) was: age = 42.55 + 7.83 * treatment condition dummy variable - 1.82 * female gender dummy variable - 3.29 * admits + 0.51 * hospitalization days - 1.40 * ER visits + 0.11 * office visits - 0.06 * other procedures + 0.15 * laboratory tests + 0.27 * radiology

visits - 0.15 * home visits + 0.11 * prescriptions - 0.00000064 * cost.

Table 7: Descriptive Summary of Variables

<u>Ordered Class Variable (Age) and Attributes</u>				
<u>Variable</u>	<u>Mean</u>	<u>SD</u>	<u>CV (%)</u>	<u>Median</u>
Age	45.58	12.04	26.43	48
Admits	0.51	0.50	98.34	1
Days	0.30	1.50	503.13	0
ER	0.20	0.64	316.15	0
Office	5.87	5.66	96.41	4
OP	9.26	12.68	136.95	4
Lab	3.08	4.02	130.86	2
Radiology	1.66	3.04	183.03	1
Home	0.03	0.51	1,796.28	0
Rx	17.30	23.04	133.21	9
Cost	4,016	7,044	175.38	2,146

Categorical Attributes

<u>Variable</u>	<u>No (0)</u>	<u>Yes (1)</u>
Treatment	1,628	374
Female	984	1,018

This MRA model explained 19.30% of the variation in patient age: $F(12,1989) = 39.6$, $p < 0.0001$. These findings indicate that a statistically significant, ecologically modest linear relationship exists between age and one or more members of the set of 12 independent variables.

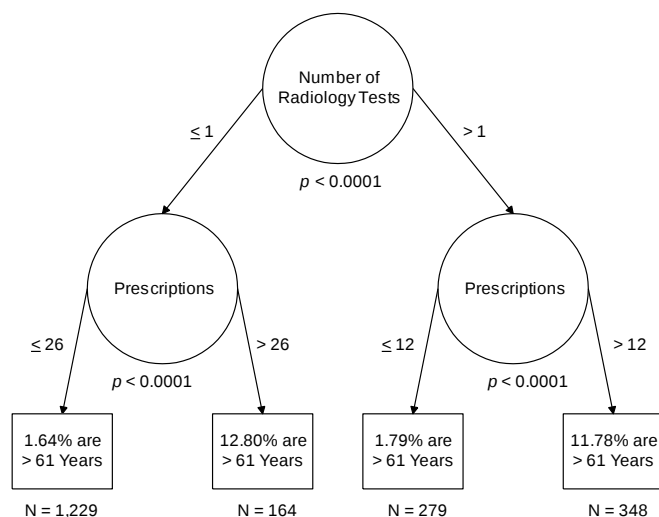
The ANOVA source table for individual variables in the MRA model (sum of squares for variable-entered-last method¹⁸) revealed statistically significant positive associations (with age) of assignment to treatment condition and number of prescriptions (p 's < 0.0001), and statistically significant negative associations (with age) of female gender ($p < 0.0003$), and number of hospital admissions ($p < 0.0021$) and ER visits ($p < 0.0149$).

Finally, considered from the perspective of predictive accuracy, for this MRA model ESS = 0.43 in training analysis—a tiny effect.

Novometric Analysis: Age dummy codes were created on the basis of statistical power analysis that indicated a minimum of 32 patients should be classified into each model endpoint to attain 90% power to detect a moderate effect. A total of 16 patients were 18 years old, and 43 were 19 years or younger, so 19 years of age was selected as the minimum age for PA. And, 31 patients were 63-64 years old, 95 were ≥ 62 , so 62 years of age was selected as the maximum age for PA. Since all of the age categories 19 to 62 inclusive were populated by data, PA created 44 dummy age class variables.

In novometric analysis the age dummy codes were treated as class variables and all other study variables were treated as attributes (assignment to treatment condition and gender are categorical).² The best (lowest D statistic, stable in LOO analysis) four-, three-, and two-strata models all emerged for an age cut-point of 61 years of age. The best four-strata model (D = 4.34) is presented in Figure 1.

Figure 1: Best Four-Strata Model



An interesting geometry underlies the model: number of prescriptions enters both the left- and right-hand branches emanating from the root attribute. On both branches “fewer” prescriptions accurately predicts that <2% of the patients are >61 years, and “more” prescriptions

accurately predicts that >11% of the patients are >61 years. Fewer and more are identified as 26 and 12 prescriptions on the left- and right-hand branches, respectively. Obviously the failure to parse prescriptions by the number of radiology tests, instead collapsing data, would mask this finding—a form of paradoxical confounding that can only be circumvented using the present methodology.^{2,19} Table 8 gives the confusion matrix for this model applied to the data.

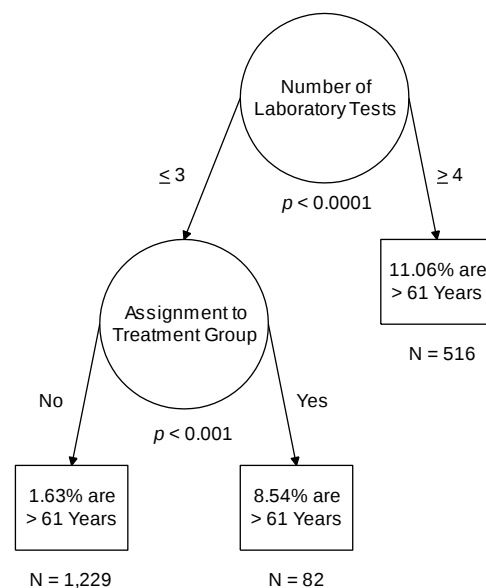
Table 8: Confusion Matrix: Four-Strata Model

		Predicted Age		
		≤61	>61	
Actual Age	≤61	1,293	450	74.18%
	>61	22	62	73.81%

The CTA model accurately predicted 3 of 4 patients who were 61 years or younger, and also 3 of 4 patients older than 61 years. The ESS of 47.99 indicates this is a high-moderate effect bordering the threshold (50) used to indicate a relatively strong effect.^{2,8}

The best three-strata model identified (D = 3.59) is presented in Figure 2.

Figure 2: Best Three-Strata Model



The three- and four-strata models use different attribute sets—they have no attributes in common. Table 9 gives the confusion matrix for the three-strata model applied to the data.

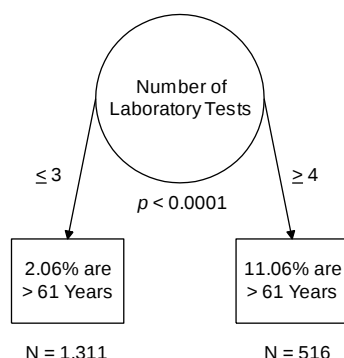
Table 9: Confusion Matrix: Three-Strata Model:

		Predicted Age		
		≤61	>61	
Actual Age	≤61	1,209	534	69.36%
	>61	20	62	76.19%

The CTA model accurately predicted 7 of 10 patients 61 years or younger, and 3 of 4 patients older than 61 years. The ESS of 45.55 indicates this effect is moderate strength.^{2,8}

Finally, the best (and globally-optimal) two-strata model identified in this application (D = 2.82) is illustrated in Figure 3.

Figure 3: Two-Strata, Globally Optimal Model



Note that while the right-hand branches of the models are identical, the left-hand branch of the two-strata model (Figure 3) integrates the patients disaggregated by the three-strata model (Figure 2) on the basis of their assignment to the control or treatment group. Thus the two-strata model has greater parsimony (it is 33.33% less complex than the three-strata model), but it also has lower predictive accuracy—in the present case, $(45.55 - 41.52) / 45.55 \times 100\% = 8.85\%$ lower ESS. Model efficiency², the ESS-to-strata ratio, is 15.18 for the three-strata model versus 20.76 for the two-strata model. Thus, compared

to the three-strata model, the two-strata model is 33% more parsimonious and $[(20.76 / 15.18) - 1] \times 100\% = 36.76\%$ more efficient, however it returns 8.85% lower predictive accuracy.

The D statistic integrates the dimensions of predictive accuracy and parsimony, thereby eliminating ambiguity concerning identification of the model that is closest (i.e., most similar) to a theoretically ideal model—physically located at the maximum orthogonal intersection of the accuracy and parsimony dimensions.² For the two-strata model, the D statistic indicates that a minimum of 2.82 additional attributes having an equivalent ESS (i.e., as was obtained for number of laboratory tests) are needed to achieve an ideal (i.e., maximum accuracy, maximum parsimony) model for this application.

The confusion matrix for this two-strata model applied to the data is given in Table 10.

Table 10: Confusion Matrix: Globally-Optimal Two-Strata Model

		Predicted Age		
		≤61	>61	
Actual Age	≤61	1,284	459	73.67%
	>61	27	57	67.86%

The CTA model accurately predicted 3 of 4 patients who were 61 years or younger, and 2 of 3 patients older than 61 years. The ESS of 41.52 reveals this is a moderate strength effect.

Comments

The literature is rich in correlation and regression results that are not statistically significant, or that while statistically significant—when considered singly in univariable designs, or together in multivariable designs—explain a functionally useless (often trivial) proportion of variance. Indeed, most published regression models are only accurate when used to predict dependent measure values at or near the sample mean.² However, as seen presently, the absence

of a strong regression result doesn't mean that there is no linear relationship between variables.

The first example featured a correlation coefficient for which the associated p -value would easily satisfy the experimentwise criterion for statistical significance in most research applications. However, this model explained an uninspiring one-eighth of the variation in the dependent variable—not accounting for the effect of chance ($ESS=1.43$). Using novometrics to explore the relationship between the variables a two-strata model emerged that produced a moderate effect ($ESS=38.95$)—correctly classifying two of three people aged 64.5 years or younger, and three of four older people.

The second example was a correlation coefficient for which the associated p -value would also easily satisfy the experimentwise criterion for statistical significance in most published research. However, this model explained a deflating one-forty-second of the variation in the dependent variable—not accounting for chance ($ESS=0.13$). Using novometrics to study the relationship between the two variables, a two-strata model was identified that produced a moderate effect ($ESS=25.67$)—correctly classifying nine in ten patients aged ≤ 62.2 years as having ≤ 22 annual office visits, and one in three older patients as having >22 annual office visits.

In the ODA laboratory such findings—which we suspect to be the rule rather than the exception, inspire our curiosity concerning the number and nature of discoveries that remain hidden, buried by debris created by the use of obsolete “data-mining” machinery. Discovering between-group effects such as those identified presently is crucial in causal inference research, for example to assess comparability of different study groups²⁰ and identify variables to consider in propensity score development.^{21,22}

The third example was a MRA model for which the associated p -value likewise would satisfy experimentwise statistical significance criteria in most applications. Using ten ordered independent variables, and two categorical

attributes, this model explained a mediocre one-fifth of the variation in the dependent variable—not accounting for the effect of chance ($ESS=0.43$). Furthermore, the conceptual meaning and translational potential of the model are unclear: statistically reliable positively weighted terms emerged for patient assignment to treatment condition and for number of prescriptions, and negatively weighted terms emerged for female gender, number of hospital admissions, and number of ER visits.

When novometrics was used to explore the relationship between age and the attributes, a single age cut-point (≤ 61 years) emerged that produced the best two- ($ESS=41.52$, $D=2.82$), three- ($ESS=45.55$, $D=3.59$), and four-strata ($ESS=47.99$, $D=4.34$) models. Application (e.g., in follow-up research) of increasingly complex models (i.e., models with an increasing number of strata) requires increasingly large training samples, because a minimum N is required in every model endpoint to achieve adequate statistical power for testing *a priori* hypotheses.² For example, it is easy to imagine how forcing a dozen attributes into an optimal model in the present study would create a blizzard of (near) empty endpoints, statistically insignificant effects, and zero-to-negative ESS values obtained in LOO analysis.^{2,23}

Finally, the third example involved both ordered and categorical attributes. Attributes may also all be categorical: such designs are traditionally analyzed using ANOVA.²⁴ The method presented herein replaces reverse CTA methodology used with these designs to identify hierarchically, but not enumerated or globally optimal CTA models.^{2, 25-27}

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Author Notes

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