

UniODA vs. Chi-Square: Deciphering $R \times C$ Contingency Tables

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Whereas omnibus effects identified using χ^2 to assess association in $R \times C$ contingency tables are often difficult or impossible to disentangle when R and C are greater than two, identifying the structure underlying $R \times C$ tables using UniODA is straightforward.¹ An investigation of the association between furniture factory production shift and type of defect is presented as an example.

Maximum-accuracy analysis of unweighted $R \times C$ tables is illustrated with an example in which a furniture company operating three production shifts categorized a total of $N = 309$ furniture defects into one of four qualitative categories² (an obvious weighted analysis would weight each defect by its associated cost, and the analysis would identify the model that explicitly minimized cost^{1,3}). The resulting 3×4 contingency table is presented in Table 1.

Table 1: Production Shift and
Type of Furniture Defect

	<u>Production Shift</u>			
<u>Defect</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>Sum</u>
A	15	26	33	74
B	21	31	17	69
C	45	34	49	128
D	13	5	20	38
Sum	94	96	119	309

The association between production shift and type of defect was assessed using chi-square

analysis.² Here, χ^2 ($df = 6$, $N = 309$) = 19.2, $p < 0.05$. Therefore the null hypothesis that there is no association is rejected, and it is concluded that an association exists between shift and type of defect.¹ This finding begs two questions: (1) what, exactly, is the structure of the association that exists between production shift and type of defect, and (2) what is the strength of this association? Disentangling this omnibus effect and assessing effect size using chi-square in this example is challenging: intrigued readers are encouraged to decipher the finding.⁴⁻⁶

The nondirectional UniODA model discriminating production shift as a function of type of furniture defect was identified using the following UniODA^{1,3} and MegaODA⁷⁻⁹ software syntax:

```
OPEN factory.dat;
OUTPUT results.out;
VARS shift defect;
CLASS shift;
ATTRIBUTE defect;
CATEGORICAL defect;
```

MCARLO ITER 25000;
GO;

The UniODA model was: if defect = A or D, then predict that shift = 3; if defect = B, then predict shift = 2; and if defect = C, then predict shift = 1. The association identified by UniODA was statistically significant ($p < 0.02$), however it reflects a relatively weak effect ($ESS = 12.4$).^{1,3} The predictive accuracy of the model is summarized in the confusion table in Table 2.

Table 2: Confusion Table for UniODA Model

Actual Shift	Predicted Shift			
	<u>1</u>	<u>2</u>	<u>3</u>	
1	45	21	28	47.9
2	34	31	31	32.3
3	49	17	53	44.5
Predictive Value	35.2	44.9	47.3	

Model sensitivities reveal that defects associated with shifts 1 and 3 are more representative of the empirical data than are findings for shift 2. The model accurately classified $45 + 31 + 53 = 129$ defects, leaving $309 - 129 = 180$ defects unexplained. Eliminating the correctly classified observations from the original data creates a residual sample: secondary structure underlying the residuals can be identified by structural decomposition analysis using the same UniODA syntax with data in the residual sample used as the input data file.^{1,3}

Statistical modeling is useful for identifying reliable effects, but in this example the effect size for the statistical model is weak, while the practical (logistical) implications of the model are complex—involving four interventions (one for each type of defect) for three shifts—all in an effort to eliminate 129 defects.

In light of the findings in Table 1, and in the absence of cost weights, a more reasonable approach presently would be to target the most likely defect—C, which accounts for 41.4% of

all defects, and is the most frequently observed defect in shift 1 (47.9%), shift 2 (35.4%), and shift 3 (41.2%). This is not only less complex (one intervention used across all shifts), but it would facilitate assessing the generalizability of the intervention (and any specific requirements) across shifts. After the intervention is completed the third row of the original data is eliminated, simplifying the problem to a 3 x 3 contingency table. Assessing the efficacy of the intervention would also enable investigators to evaluate the possible interaction of the intervention with respect to the distribution of defects of all four qualitative types.

References

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Author Notes

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