

# UniODA vs. Legacy Bivariate Statistical Methodologies

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Research comparing the use of optimal versus legacy methods for analysis of data representing different experimental designs is on-going. This note discusses bivariate legacy statistical tools for which the alternative use of UniODA has already been demonstrated as an always valid, exact, maximum-accuracy statistical methodology.

At first blush univariate optimal discriminant analysis, called UniODA<sup>1,2</sup> is an inauspiciously compact decision-making algorithm with only one objective function—to identify the most accurate classification model for any given sample of classical data. Classification models that are developed using UniODA define the statistical contextual meaning of the words “transparent” and “parsimony”.<sup>1-5</sup> Created using a “training sample” a UniODA model predicts the state of the class (“dependent”) variable for individual observations, on the basis of the value of one or more attributes (“independent variables”).<sup>1</sup>

For clarity, imagine that the binary class variable is gender, having two class categories. Indicator values<sup>6</sup> used in class coding are arbitrary: imagine that for males, class = 1, and for females, class = 0.

For an ordered attribute, a UniODA model finds the *threshold* that yields greatest accuracy—normed against chance—for the sample: if score  $\leq$  *threshold*, then predict that the observation is from class 1; otherwise predict that the observation is from class 0.

For a categorical attribute, a UniODA model finds the *mapping* from attribute to class

variable that yields greatest (weighted) accuracy—normed against chance—for the sample: if score = *category list*, then predict that the observation is from class 1; otherwise predict that the observation is from class 0.

Regardless of the nature of the metric used in attribute measurement, in UniODA any problem can be weighted.<sup>1</sup> In an experimental context, if a weight is used conceptually to define the desired objective function, then the empirical failure to weight the problem is a crucial limitation of the experimental design.<sup>1</sup> For example, in a study of weight loss, if the observations in the training sample (that is used to construct the model) are *unit-weighted* (i.e., are assigned equivalent weight), then UniODA will identify the model that maximizes accuracy in predicting whether or not an individual observation lost weight over the course of the study. However, if observations are instead weighted by the number of pounds that they added or subtracted over the course of the study, then weighted UniODA will identify the model that maximizes accuracy in predicting the number of pounds an individual observation lost or gained over the course of the study. Any quantitative

weight can be maximized, and synergistic interactions of quantitative weights can be maximized. For example, if it is appropriate in a study to weight observations by the amount of force that they expended, then the product of acceleration times mass would be appropriate as a measure of force by which to weight observations, and accuracy of the model in predicting expended force would be maximized.

While research comparing optimal versus legacy methods for analyzing data from different experimental designs is on-going, the present note summarizes bivariate legacy statistical tools for which the alternative use of UniODA has already been demonstrated as a valid, exact, transparent, maximum-accuracy, alternative statistical methodology.

### Initial and Early Research

When the algorithm and software system were first introduced, it was already clear that UniODA was powerful—obtaining more accurate and parsimonious models, in wide domains of substantive application and experimental design, than were obtained using legacy statistical methods and software.<sup>1,7-10</sup> And, it also was already clear that the UniODA algorithm not only offers advantages, but it also overcomes well-known limitations of legacy tools.<sup>11,12</sup> For example, unlike UniODA, legacy tools *don't* explicitly maximize *the accuracy of the model for the data*: instead, legacy methods maximize other objective functions (e.g., variance ratios or the value of the likelihood function), and make distributional assumptions *with which data must comply* in order for the method to produce valid results.<sup>3-5</sup> An advantage of UniODA is that the level of classification accuracy that is achieved by an “optimal” (maximum-accuracy) model<sup>13</sup> is summarized using the normed effect strength for sensitivity (ESS) statistic: 0 is the classification accuracy expected by chance for the application and sample being analyzed, and 100 is perfect, errorless classification of all observations in the sample.<sup>1</sup> The ESS statistic enables

direct, meaningful comparison of different models with respect to a standardized measure of additional classification above and beyond what is achieved by chance.<sup>1,14-16</sup>

Initial research involved comparing UniODA versus *t*-test.<sup>1-5,17</sup> This was generalized to more complex general linear model designs. For example, UniODA was found to be a superior alternative to one-way<sup>18,19</sup> and factorial<sup>20</sup> analysis of variance, and to analysis of covariance designed to control (eliminate) the effect of a confounding variable.<sup>21</sup> Early research also investigated the use of UniODA to improve the accuracy of linear models derived using legacy methods such as logistic regression analysis, Fisher's linear discriminant analysis, log-linear analysis, and probit analysis.<sup>1,22-24</sup> UniODA-based optimization methods were expanded to include ordinary least-squares (multiple) regression.<sup>25-28</sup> While UniODA offered substantive advantages versus legacy methods—such as exact *p*-values, and maximum classification accuracy normed against chance, UniODA also eliminated issues associated with use of legacy methods—such as evaluating crucial distributional assumptions (and handling violations) underlying the validity of legacy models, model sensitivity to outlying data, and regression toward the mean.

Other early research explored the development of optimal linear multivariable models, found to be more accurate and more parsimonious than sub-optimal legacy linear models.<sup>29-33</sup> However, optimal nonlinear classification tree analysis<sup>34-37</sup> conducted for ordered class variables<sup>38</sup> identifies significantly more accurate and parsimonious models than are obtained by optimized (multiple) regression.

### Evolving State-of-the-Art

Additional examples of advantages of UniODA—representing solutions to problems incurred by legacy methods—emerged for chi-square analysis<sup>39-41</sup>, the log-linear model<sup>42</sup> and logistic regression analysis.<sup>43-45</sup> While UniODA

offers substantive advantages versus legacy methods—such as exact, confirmatory as well as exploratory  $p$ -values, and yielding maximum (weighted) classification accuracy normed for chance, UniODA also eliminates sticky issues associated with use of legacy methods—such as evaluating and dealing with violations of crucial distributional assumptions underlying validity, model sensitivity to skewed and/or tied data, use of dummy-variables to decompose multicategorical attributes and associated design matrix hyper-growth, and the inherent susceptibility of all linear models to paradoxical confounding.

UniODA has also been demonstrated as a superior test of null hypotheses conventionally evaluated using an expanding variety of legacy statistical tools, such as polychoric correlation<sup>46</sup>, weighted<sup>47</sup> or unweighted<sup>48</sup> kappa, ROC analysis<sup>49</sup>, the Bray-Curtis dissimilarity test<sup>50</sup>, Bowker's test for symmetry<sup>51</sup>, Kendall's coefficient of concordance<sup>52</sup>, McNemar's test for correlated proportions<sup>53,54</sup>, Mann-Whitney  $U$  test,<sup>55,56</sup> and both inter-method and inter-rater reliability.<sup>57-59</sup> Advantages of UniODA versus legacy methods include exact exploratory and confirmatory  $p$ -values, and transparent models expressed in original measurement units, that explicitly achieve maximum (weighted) classification accuracy normed against chance. UniODA also eliminates troublesome issues associated with use of legacy methods—such as evaluating and dealing with violations of crucial distributional assumptions underlying validity, model sensitivity to outlying, skewed and/or tied data, use of dummy-variables to decompose multicategorical attributes and associated design matrix hyper-growth, regression toward the mean, and the inherent susceptibility of all linear models to paradoxical confounding.

Finally, conceptually related to early work considering optimal analysis of temporal phenomena as represented in turnover tables and in Markov models of state transitions<sup>1</sup>, recent research has extended temporal analysis to the

study of all ordered series, including single-case longitudinal series.<sup>60-65</sup>

This appears to be only the beginning of the story regarding the ability of the unassuming UniODA algorithm to supplant legacy statistical methods—in situations in which the objective function being maximized is the accuracy and parsimony of empirical models.

## References

- <sup>1</sup>Yarnold PR, Soltysik RC (2005). *Optimal data analysis: A guidebook with software for Windows*, Washington, DC, APA Books.
- <sup>2</sup>Yarnold PR, Soltysik RC (2010). Optimal data analysis: A general statistical analysis paradigm. *Optimal Data Analysis*, 1, 10-22. URL: <http://odajournal.com/2013/09/19/optimal-data-analysis-a-general-statistical-analysis-paradigm/>
- <sup>3</sup>Yarnold PR, Soltysik RC (1991). Theoretical distributions of optima for univariate discrimination of random data. *Decision Sciences*, 22, 739-752. DOI: 10.1111/j.1540-5915.1991.tb00362
- <sup>4</sup>Soltysik RC, Yarnold PR (1994). Univariable optimal discriminant analysis: One-tailed hypotheses. *Educational and Psychological Measurement*, 54, 646-653. DOI: 10.1177/0013164494054003007
- <sup>5</sup>Carmony L, Yarnold PR, Naeymi-Rad F (1998). One-tailed Type I error rates for balanced two-category UniODA with a random ordered attribute. *Annals of Operations Research*, 74, 223-238. DOI: 10.1023/A:1018922421450
- <sup>6</sup>Bryant FB, Harrison PR (2013). How to create an ASCII input data file for UniODA and CTA Software. *Optimal Data Analysis*, 3, 2-6. URL: <http://odajournal.com/2013/11/04/how-to-create-a-data-set-with-sas-and-compare-attributes-with-unioda-in-serial-single-case-designs/>
- <sup>7</sup>Soltysik RC, Yarnold PR (2013). MegaODA large sample and BIG DATA time trials: Separating the chaff. *Optimal Data Analysis*, 2, 194-197. URL: <http://odajournal.com/2013/11/19/megaoda-large-sample-and-big-data-time-trials-separating-the-chaff/>

- <sup>8</sup>Soltysik RC, Yarnold PR (2013). MegaODA large sample and BIG DATA time trials: Harvesting the Wheat. *Optimal Data Analysis*, 2, 202-205. URL: <http://odajournal.com/2013/11/21/megaoda-large-sample-and-big-data-time-trials-harvesting-the-wheat/>
- <sup>9</sup>Yarnold PR, Soltysik RC (2013). MegaODA large sample and BIG DATA time trials: Maximum velocity analysis. *Optimal Data Analysis*, 2, 220-221. URL: <http://odajournal.com/2013/11/27/megaoda-large-sample-and-big-data-time-trials-maximum-velocity-analysis/>
- <sup>10</sup>Bryant FB (2010). The Loyola experience (1993-2009): Optimal data analysis in the Department of Psychology. *Optimal Data Analysis*, 1, 4-9. URL: <http://odajournal.com/2013/09/19/the-loyola-experience-1993-2009-optimal-data-analysis-in-the-department-of-psychology/>
- <sup>11</sup>Grimm LG, Yarnold PR (Eds.). *Reading and understanding multivariate statistics*. Washington, D.C.: APA Books, 1995.
- <sup>12</sup>Grimm LG, Yarnold PR (Eds.). *Reading and understanding more multivariate statistics*. Washington, D.C.: APA Books, 2000.
- <sup>13</sup>Yarnold PR (2014). "A statistical guide for the ethically perplexed" (Chapter 4, Panter & Sterba, *Handbook of Ethics in Quantitative Methodology*, Routledge, 2011): Clarifying disorientation regarding the etiology and meaning of the term *Optimal* as used in the Optimal Data Analysis (ODA) paradigm. *Optimal Data Analysis*, 3, 30-31. URL: <http://odajournal.com/2014/04/06/a-statistical-guide-for-the-ethically-perplexed-chapter-4-panter-sterba-handbook-of-ethics-in-quantitative-methodology-routledge-2011-clarifying-disorientation-regarding/>
- <sup>14</sup>Yarnold PR (2013). Minimum standards for reporting UniODA findings. *Optimal Data Analysis*, 2, 63-68. URL: <http://odajournal.com/2013/10/14/first-real-time-oda-article-published-in-volume-2/>
- <sup>15</sup>Yarnold PR (2013). Minimum standards for reporting UniODA findings for class variables with three or more response categories. *Optimal Data Analysis*, 2, 86-93. URL: <http://odajournal.com/2013/10/22/minimum-standards-for-reporting-unioda-findings-for-class-variables-with-three-or-more-response-categories/>
- <sup>16</sup>Yarnold PR (2013). Standards for reporting UniODA findings expanded to include ESP and all possible aggregated confusion tables. *Optimal Data Analysis*, 2, 106-119. URL: <http://odajournal.com/2013/10/29/standards-for-reporting-unioda-findings-expanded-to-include-esp-and-all-possible-aggregated-confusion-tables/>
- <sup>17</sup>Yarnold PR (2014). UniODA vs. *t*-Test: Comparing two migraine treatments. *Optimal Data Analysis*, 3, 6-8. URL: <http://odajournal.com/2014/03/27/unioda-vs-students-t-test-comparing-two-migraine-treatments-2/>
- <sup>18</sup>Yarnold PR (2013). ODA range test vs. one-way analysis of variance: Patient race and lab results. *Optimal Data Analysis*, 2, 206-210. URL: <http://odajournal.com/2013/11/22/oda-range-test-vs-one-way-analysis-of-variance-patient-race-and-lab-results/>
- <sup>19</sup>Yarnold PR, Brofft GC (2013). Comparing knot strength with UniODA. *Optimal Data Analysis*, 2, 54-59. URL: <http://odajournal.com/2013/11/19/oda-range-test-vs-one-way-analysis-of-variance-comparing-strength-of-alternative-line-connections/>
- <sup>20</sup>Yarnold PR, Soltysik RC (2013). Reverse CTA: An optimal analog to analysis of variance. *Optimal Data Analysis*, 2, 43-47. URL: <http://odajournal.com/2013/09/20/reverse-cta-versus-multiple-regression-analysis/>
- <sup>21</sup>Yarnold PR (2015). Evaluating non-confounded association of an attribute and a class variable using partial UniODA. *Optimal Data Analysis*, 4, 32-35. URL: <http://odajournal.com/2015/05/04/evaluating-non-confounded-association-of-an-attribute-and-a-class-variable-using-partial-unioda/>
- <sup>22</sup>Yarnold PR, Soltysik RC (1991). Refining two-group multivariable classification models using univariate optimal discriminant analysis. *Decision Sciences*, 22, 1158-1164. DOI: 10.1111/j.1540-5915.1991.tb01912.x

- <sup>23</sup>Yarnold PR, Hart LA, Soltysik RC (1994). Optimizing the classification performance of logistic regression and Fisher's discriminant analyses. *Educational and Psychological Measurement*, 54, 73-85. DOI: 10.1177/0013164494054001007
- <sup>24</sup>Yarnold BM, Yarnold PR (2010). Maximizing the accuracy of Probit models via UniODA. *Optimal Data Analysis*, 1, 41-42. URL: <http://odajournal.com/2013/09/19/maximizing-the-accuracy-of-probit-models-via-unioda/>
- <sup>25</sup>Yarnold PR, Bryant FB, Soltysik RC (2013). Maximizing the accuracy of multiple regression models via UniODA: Regression *away from* the mean. *Optimal Data Analysis*, 2, 19-25. URL: <http://odajournal.com/2013/09/20/maximizing-the-accuracy-of-multiple-regression-models-using-unioda-regression-away-from-the-mean/>
- <sup>26</sup>Yarnold PR (2013). Maximum-accuracy multiple regression analysis: Influence of registration on overall satisfaction ratings of emergency room patients. *Optimal Data Analysis*, 2, 72-75. URL: <http://odajournal.com/2013/10/17/maximum-accuracy-multiple-regression-analysis-influence-of-registration-on-overall-satisfaction-ratings-of-emergency-room-patients/>
- <sup>27</sup>Yarnold PR (2013). Assessing technician, nurse, and doctor ratings as predictors of overall satisfaction ratings of Emergency Room patients: A maximum-accuracy multiple regression analysis. *Optimal Data Analysis*, 2, 76-85. URL: <http://odajournal.com/2013/10/21/assessing-technician-nurse-and-doctor-ratings-as-predictors-of-overall-satisfaction-of-emergency-room-patients-a-maximum-accuracy-multiple-regression-analysis/>
- <sup>28</sup>Yarnold PR (2015). Maximizing ESS of regression models in applications with dependent measures with domains exceeding ten values. *Optimal Data Analysis*, 4, 12-13. URL: <http://odajournal.com/2015/01/17/maximizing-ess-of-regression-models-in-applications-with-dependent-measures-with-domains-exceeding-ten-values/>
- <sup>29</sup>Yarnold PR, Soltysik RC, Martin GJ (1994). Heart rate variability and susceptibility for sudden cardiac death: An example of multivariable optimal discriminant analysis. *Statistics in Medicine*, 13, 1015-1021. DOI: 10.1002/sim.4780131004
- <sup>30</sup>Soltysik RC, Yarnold PR (1994). The Warmack-Gonzalez algorithm for linear two-category multivariable optimal discriminant analysis. *Computers and Operations Research*, 21, 735-745. DOI: 10.1016/0305-0548(94)90003-5
- <sup>31</sup>Yarnold PR, Soltysik RC, McCormick WC, Burns R, Lin EHB, Bush T, Martin GJ (1995). Application of multivariable optimal discriminant analysis in general internal medicine. *Journal of General Internal Medicine*, 10, 601-606. DOI: 10.1007/BF02602743
- <sup>32</sup>Yarnold PR, Soltysik RC, Lefevre F, Martin GJ (1998). Predicting in-hospital mortality of patients receiving cardiopulmonary resuscitation: Unit-weighted MultiODA for binary data. *Statistics in Medicine*, 17, 2405-2414. DOI: 10.1002/(SICI)1097-0258
- <sup>33</sup>Soltysik RC, Yarnold PR (2010). Two-group' MultiODA: Mixed-integer linear programming solution with bounded *M*. *Optimal Data Analysis*, 1, 31-37. URL: <http://odajournal.com/2013/09/19/two-group-multioda-a-mixed-integer-linear-programming-solution-with-bounded-m/>
- <sup>34</sup>Soltysik RC, Yarnold PR (2010). Automated CTA software: Fundamental concepts and control commands. *Optimal Data Analysis*, 1, 144-160. URL: <http://odajournal.com/2013/09/19/62/>
- <sup>35</sup>Yarnold PR, Bryant FB (2015). Obtaining a hierarchically optimal CTA model via UniODA software. *Optimal Data Analysis*, 4, 36-53.. URL: <http://odajournal.com/2015/05/14/obtaining-an-enumerated-cta-model-via-automated-cta-software-2/>



- <sup>36</sup>Yarnold PR (2015). Obtaining an enumerated CTA model via automated CTA software. *Optimal Data Analysis*, 4, 54-60. URL: <http://odajournal.com/2015/05/14/obtaining-an-enumerated-cta-model-via-automated-cta-software-2/>
- <sup>37</sup>Yarnold PR (2013). Initial use of hierarchically optimal classification tree analysis in medical research. *Optimal Data Analysis*, 2, 7-18. URL: <http://odajournal.com/2013/09/20/initial-use-of-hierarchically-optimal-classification-tree-analysis-in-medical-research/>
- <sup>38</sup>Yarnold PR, Soltysik RC (2014). Globally optimal statistical classification models, II: Unrestricted class variable, two or more attributes. *Optimal Data Analysis*, 3, 78-84. URL: <http://odajournal.com/2014/08/25/globally-optimal-statistical-models-ii-unrestricted-class-variable-two-or-more-attributes/>
- <sup>39</sup>Yarnold PR (2010). UniODA vs. chi-square: Ordinal data sometimes feign categorical. *Optimal Data Analysis*, 1, 62-65. URL: <http://odajournal.com/2013/09/19/unioda-vs-chi-square-ordinal-data-sometimes-feign-categorical/>
- <sup>40</sup>Yarnold PR (2014). UniODA vs. chi-square: Audience effect on smile production in infants. *Optimal Data Analysis*, 3, 3-5. URL: <http://odajournal.com/2014/03/26/unioda-vs-chi-square-audience-effect-on-smile-production-in-infants/>
- <sup>41</sup>Yarnold PR (2014). UniODA vs. chi-square: Discriminating inhibited and uninhibited infant profiles. *Optimal Data Analysis*, 3, 9-11. URL: <http://odajournal.com/2014/03/27/unioda-vs-chi-square-discriminating-inhibited-and-uninhibited-infant-profiles/>
- <sup>42</sup>Yarnold PR (2010). GenUniODA vs. log-linear model: Modeling discrimination in organizations. *Optimal Data Analysis*, 1, 59-61. URL: <http://odajournal.com/2013/09/19/gen-unioda-vs-log-linear-model-modeling-organizational-discrimination/>
- <sup>43</sup>Yarnold PR (2014). UniODA vs. logistic regression analysis: Serum cholesterol and coronary heart disease and mortality among middle aged diabetic men. *Optimal Data Analysis*, 3, 17-18. URL: <http://odajournal.com/2014/03/29/unioda-vs-logistic-regression-serum-cholesterol-and-coronary-heart-disease-and-mortality-among-middle-aged-diabetic-men/>
- <sup>44</sup>Yarnold PR (2013). Analyzing categorical attributes having many response categories. *Optimal Data Analysis*, 2, 172-176. URL: <http://odajournal.com/2013/11/08/analyzing-categorical-attributes-having-many-response-options/>
- <sup>45</sup>Yarnold PR (2013). Univariate and multivariate analysis of categorical attributes with many response categories. *Optimal Data Analysis*, 2, 177-190. URL: <http://odajournal.com/2013/11/11/univariate-and-multivariate-analysis-of-categorical-attributes-with-many-response-categories/>
- <sup>46</sup>Yarnold PR (2014). UniODA vs. polychoric correlation: Number of lambs born over two years. *Optimal Data Analysis*, 3, 113-114. URL: <http://odajournal.com/2014/10/25/unioda-vs-polychoric-correlation-number-of-lambs-born-over-two-years/>
- <sup>47</sup>Yarnold PR (2014). UniODA vs. weighted kappa: Evaluating concordance of clinician and patient ratings of the patient's physical and mental health functioning. *Optimal Data Analysis*, 3, 12-13. URL: <http://odajournal.com/2014/03/28/unioda-vs-weighted-kappa-evaluating-concordance-of-clinician-and-patient-ratings-of-the-patients-physical-and-mental-health-functioning/>
- <sup>48</sup>Yarnold PR (2014). UniODA vs. kappa: Evaluating the long-term (27-year) test-retest reliability of the Type A Behavior Pattern. *Optimal Data Analysis*, 3, 14-16. URL: <http://odajournal.com/2014/03/29/unioda-vs-kappa-evaluating-the-long-term-27-year-test-retest-reliability-of-the-type-a-behavior-pattern/>
- <sup>49</sup>Yarnold PR (2014). UniODA vs. ROC analysis: Computing the "optimal" cut-point. *Optimal Data Analysis*, 3, 117-120. URL: <http://odajournal.com/2014/11/14/unioda-vs-roc-analysis-computing-the-optimal-cut-point/>
- <sup>50</sup>Yarnold PR (2014). UniODA vs. Bray-Curtis dissimilarity index for count data. *Optimal Data Analysis*, 3, 115-116. URL: <http://odajournal.com/2014/11/09/unioda-vs-bray-curtis-dissimilarity-index-for-count-data/>
- <sup>51</sup>Yarnold PR (2015). UniODA vs. Bowker's test for symmetry: Diagnosis before vs. after treatment. *Optimal Data Analysis*, 4, 29-31. URL: <http://odajournal.com/2015/04/24/unioda-vs-bowkers-test-for-symmetry-diagnosis-before-vs-after-treatment/>

<sup>52</sup>Yarnold PR (2014). UniODA vs. Kendall's Coefficient of Concordance (W): Multiple rankings of multiple movies. *Optimal Data Analysis*, 3, 121-123. URL:

<http://odajournal.com/2014/11/23/unioda-vs-kendalls-coefficient-of-concordance-w-multiple-rankings-of-multiple-movies-2/>

<sup>53</sup>Yarnold PR (2015). UniODA vs. McNemar's test for correlated proportions: Diagnosis of disease before vs. after treatment. *Optimal Data Analysis*, 4, 24-26. URL:

<http://odajournal.com/2015/04/22/unioda-vs-mcnemars-test-for-correlated-proportions-diagnosis-of-disease-before-vs-after-treatment/>

<sup>54</sup>Yarnold PR (2015). UniODA vs. McNemar's test: A small sample analysis. *Optimal Data Analysis*, 4, 27-28. URL:

<http://odajournal.com/2015/04/22/unioda-vs-mcnemars-test-a-small-sample-analysis/>

<sup>55</sup>Yarnold PR (2014). UniODA vs. Mann-Whitney U test: Sunlight and petal width. *Optimal Data Analysis*, 4, 3-5. URL:

<http://odajournal.com/2014/12/09/unioda-vs-mann-whitney-u-test-sunlight-and-petal-width/>

<sup>56</sup>Yarnold PR (2014). UniODA vs. Mann-Whitney U test: Comparative effectiveness of laxatives. *Optimal Data Analysis*, 4, 6-8. URL:

<http://odajournal.com/2014/12/27/unioda-vs-mann-whitney-u-test-comparative-effectiveness-of-laxatives-3/>

<sup>57</sup>Yarnold PR (2014). How to assess inter-observer reliability of ratings made on ordinal scales: Evaluating and comparing the Emergency Severity Index (Version 3) and Canadian Triage Acuity Scale. *Optimal Data Analysis*, 3, 42-49. URL:

<http://odajournal.com/2014/04/14/how-to-assess-the-inter-method-parallel-forms-reliability-of-ratings-made-on-ordinal-scales-emergency-severity-index-version-3-and-canadian-triage-acuity-scale/>

<sup>58</sup>Yarnold PR (2014). How to assess inter-observer reliability of ratings made on ordinal scales: Evaluating and comparing the Emergency Severity Index (Version 3) and Canadian Triage Acuity Scale. *Optimal Data Analysis*, 3, 42-49. URL:

<http://odajournal.com/2014/04/13/how-to-assess-inter-observer-reliability-of-ratings-made-on-ordinal-scales-evaluating-and-comparing-the-emergency-severity-index-version-3-and-canadian-triage-acuity-scale/>

<sup>59</sup>Yarnold PR (2015). Estimating inter-rater reliability using pooled data induces paradoxical confounding: An example involving Emergency Severity Index triage ratings. *Optimal Data Analysis*, 4, 21-23. URL:

<http://odajournal.com/2015/04/21/estimating-inter-rater-reliability-using-pooled-data-induces-paradoxical-confounding-an-example-involving-emergency-severity-index-triage-ratings/>

<sup>60</sup>Yarnold PR (2013). Statistically significant increases in crude mortality rate of North Dakota counties occurring after massive environmental usage of toxic chemicals and biocides began there in 1998: An optimal static statistical map. *Optimal Data Analysis*, 2, 98-105. URL:

<http://odajournal.com/2013/10/26/statistically-significant-increases-in-crude-mortality-rate-of-north-dakota-counties-occurring-after-massive-environmental-usage-of-toxic-chemicals-and-biocides-began-there-in-1998-an-optimal-static-2/>

<sup>61</sup>Yarnold PR (2013). Determining when annual crude mortality rate most recently began increasing in North Dakota counties, I: Backward-stepping little jiffy. *Optimal Data Analysis*, 2, 217-219. URL:

<http://odajournal.com/2013/11/24/determining-when-annual-crude-mortality-rate-most-recently-began-increasing-in-north-dakota-counties-i-backward-stepping-little-jiffy/>

<sup>62</sup>Yarnold PR (2013). The most recent, earliest, and Kth significant changes in an ordered series: Traveling backwards in time to assess when annual crude mortality rate most recently began increasing in McLean County, North Dakota. *Optimal Data Analysis*, 2, 143-147. URL:

<http://odajournal.com/2013/11/01/the-most-recent-earliest-and-kth-significant-changes-in-an-ordered-series-traveling-backwards-in-time-to-assess-when-annual-crude-mortality-rate-most-recently-began-increasing-in-mclean-county-nor/>

<sup>63</sup>Yarnold PR (2013). Comparing attributes measured with "identical" Likert-type scales in single-case designs with UniODA. *Optimal Data Analysis*, 2, 148-153. URL:

<http://odajournal.com/2013/11/02/comparing-attributes-measured-with-identical-likert-type-scales-in-single-case-designs-with-unioda/>

<sup>64</sup>Yarnold PR (2013). Comparing responses to dichotomous attributes in single-case designs. *Optimal Data Analysis*, 2, 154-156. URL:

<http://odajournal.com/2013/11/02/comparing-responses-to-dichotomous-attributes-in-single-case-designs/>

<sup>65</sup>Yarnold PR (2013). Ascertaining an individual patient's *symptom dominance hierarchy*: Analysis of raw longitudinal data induces Simpson's Paradox. *Optimal Data Analysis*, 2, 159-171. URL: <http://odajournal.com/2013/11/07/ascertaining-an-individual-patients-symptom-dominance-hierarchy-analysis-of-raw-longitudinal-data-induces-simpsons-paradox/>

### **Author Notes**

This study involved secondary data analysis of published de-identified data and was exempt from Institutional Review Board review.

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