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Statistical Power of Optimal Discrimination with a Normal Attribute and Two Classes: One-Tailed Hypotheses

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This note reports statistical power (1-β) obtained by ODA when used with a normally-distributed attribute, as a function of alpha and effect size.

Investigators in all quantitative disciplines are often interested in discovering if statistically significant differences exist between two classes with respect to a single continuous attribute. Methods for performing this assessment in the absence of distributional assumptions about the attribute under investigation have been developed.[1-4] For every application in which it is used, optimal discriminant analysis (ODA) determines a model—consisting of a cut-point (decision threshold) and a direction of classification assignment—which maximizes the number of correct classifications of the class variable along the single dimension of the attribute.[1] Exact and Monte Carlo estimates of Type I error (α) have been developed.[1-4]

Because it is metric-free, no research has yet investigated statistical power of ODA for an attribute which has been sampled from a known distribution. Indeed, a major advantage of ODA is it requires no assumptions concerning parent distributions, because data samples consistent with parametric assumptions are rarely seen in the literature.[4] Representing the first exploration of this matter, this note reports statistical power (1-β) which is obtained by ODA when used with a normally-distributed attribute—as is assumed by both the general linear model and maximum-likelihood paradigms.[4-6]

Presently the direction of classification assignment is fixed, corresponding to the evaluation of a one-tailed null hypothesis.[2] Tables 1 and 2 summarize the results of a Monte Carlo analysis for α = 0.01 and 0.05, in which 100,000 sets of normal deviates were generated for each of the two classes.[7] The membership of each of the classes is equal ($n_0 = n_1$). Power is represented by the proportion of rejections of the null hypothesis for each experiment at the corresponding level of α. Its complement β (Type II error) is the proportion of acceptances of these hypotheses. Table entries correspond to 14 values of Cohen's d, increasing values of which cause increasing separation between the observations comprising each class.[8]

These Tables may be used to estimate the sample size required to achieve a specific level of power for a given effect. For example, in a study with $n_0 = n_1 = 60$, for power of at least 90%, an effect having Cohen's d = 0.8 is required for α = 0.01, versus Cohen's d = 0.7 for α = 0.05.

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Table 1

Statistical Power of Optimal Discrimination with One Normal Attribute and Two Classes, Obtained from Monte Carlo Analysis: One-Tailed Test, α = 0.01

	Cohen's *d*													
n_1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3	1.4	1.5
5	.008	.010	.015	.021	.028	.036	.045	.059	.074	.091	.112	.136	.161	.192
6	.002	.004	.005	.008	.011	.015	.022	.029	.037	.051	.063	.080	.101	.125
7	.009	.014	.020	.029	.039	.054	.073	.093	.121	.151	.188	.231	.273	.322
8	.021	.031	.044	.062	.084	.112	.147	.188	.232	.285	.340	.398	.461	.524
9	.008	.013	.019	.029	.043	.062	.084	.114	.147	.191	.241	.294	.353	.416
10	.016	.025	.038	.055	.079	.110	.147	.195	.246	.306	.371	.441	.509	.583
11	.006	.011	.017	.028	.043	.064	.087	.123	.165	.213	.273	.338	.408	.477
12	.011	.019	.031	.049	.071	.103	.143	.193	.252	.317	.391	.469	.546	.621
13	.019	.031	.047	.074	.109	.152	.209	.273	.345	.424	.508	.588	.666	.739
14	.028	.044	.070	.103	.150	.209	.275	.354	.437	.523	.610	.691	.760	.825
15	.013	.023	.037	.061	.094	.137	.191	.262	.336	.423	.509	.599	.681	.756
16	.019	.033	.053	.084	.130	.186	.249	.336	.421	.514	.606	.689	.767	.831
17	.026	.044	.073	.114	.166	.236	.316	.404	.502	.596	.689	.767	.834	.889
18	.014	.024	.042	.070	.110	.163	.230	.312	.404	.504	.599	.690	.772	.840
19	.017	.033	.056	.092	.142	.207	.287	.377	.481	.586	.678	.763	.835	.889
20	.024	.042	.073	.118	.178	.254	.346	.449	.554	.654	.745	.822	.884	.927
21	.030	.055	.093	.145	.218	.304	.404	.513	.620	.717	.800	.867	.917	.951
22	.016	.031	.055	.097	.153	.225	.314	.419	.530	.633	.735	.813	.878	.925
23	.021	.039	.071	.117	.181	.270	.368	.481	.592	.696	.788	.859	.913	.949
24	.026	.049	.088	.144	.220	.315	.421	.540	.652	.749	.833	.895	.939	.966
25	.032	.059	.104	.170	.257	.361	.476	.592	.703	.798	.872	.923	.956	.978
30	.038	.075	.133	.215	.322	.448	.579	.699	.803	.882	.934	.967	.984	.993
40	.044	.092	.172	.286	.430	.579	.719	.830	.911	.958	.982	.993	.998	.999
50	.046	.103	.199	.337	.505	.672	.809	.902	.958	.984	.995	.998	.999	
60	.067	.153	.289	.466	.651	.803	.908	.965	.989	.997	.999			
70	.091	.202	.370	.570	.754	.885	.956	.988	.997	.999				
80	.079	.193	.371	.583	.778	.904	.967	.991	.998	.999				
90	.096	.232	.437	.661	.840	.943	.985	.997	.999					
100	.112	.269	.498	.727	.888	.966	.993	.998	.999					

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Table 2

Statistical Power of Optimal Discrimination with One Normal Attribute and Two Classes, Obtained from Monte Carlo Analysis: One-Tailed Test, $\alpha = 0.05$

n_1	Cohen's *d*													
	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3	1.4	1.5
4	.024	.031	.040	.053	.063	.082	.098	.117	.139	.166	.194	.222	.256	.293
5	.066	.086	.107	.138	.166	.200	.237	.284	.325	.372	.424	.474	.524	.576
6	.025	.034	.047	.063	.084	.105	.135	.166	.201	.246	.285	.337	.388	.438
7	.052	.070	.093	.124	.156	.198	.244	.294	.348	.406	.465	.530	.587	.643
8	.085	.115	.149	.194	.241	.297	.360	.422	.487	.557	.618	.680	.739	.787
9	.038	.054	.077	.104	.142	.185	.234	.293	.354	.421	.490	.558	.628	.687
10	.059	.083	.117	.157	.209	.265	.328	.401	.472	.545	.616	.687	.749	.802
11	.081	.116	.162	.214	.274	.346	.421	.500	.578	.652	.721	.784	.839	.881
12	.110	.152	.209	.271	.341	.425	.508	.588	.670	.739	.804	.857	.897	.931
13	.056	.084	.122	.171	.235	.303	.384	.467	.551	.635	.711	.778	.838	.884
14	.074	.111	.159	.219	.291	.374	.458	.552	.636	.713	.786	.847	.892	.928
15	.093	.140	.196	.270	.348	.440	.531	.626	.706	.784	.844	.894	.931	.957
16	.117	.171	.236	.319	.413	.505	.596	.690	.770	.837	.890	.928	.955	.973
17	.064	.101	.153	.217	.297	.390	.488	.583	.677	.757	.829	.884	.926	.956
18	.081	.125	.184	.261	.350	.449	.549	.647	.737	.815	.875	.920	.951	.973
19	.096	.148	.219	.304	.400	.501	.606	.704	.789	.857	.907	.944	.969	.983
20	.113	.174	.252	.346	.451	.560	.661	.754	.833	.892	.934	.962	.980	.990
21	.131	.199	.287	.387	.498	.608	.707	.798	.868	.918	.952	.973	.987	.993
22	.079	.132	.201	.290	.394	.502	.612	.716	.804	.872	.924	.955	.977	.988
23	.092	.150	.231	.326	.434	.553	.664	.763	.843	.903	.943	.969	.984	.992
24	.108	.172	.260	.365	.481	.598	.706	.801	.873	.924	.959	.979	.990	.995
25	.121	.194	.285	.400	.523	.640	.749	.832	.898	.944	.970	.985	.993	.997
30	.125	.206	.315	.440	.570	.698	.805	.883	.936	.967	.985	.994	.997	.999
40	.185	.304	.451	.604	.746	.853	.924	.966	.986	.995	.998	.999		
50	.167	.294	.452	.627	.775	.885	.949	.980	.994	.998	.999			
60	.201	.359	.546	.723	.858	.937	.979	.994	.998	.999				
70	.233	.410	.614	.790	.908	.967	.990	.998	.999					
80	.255	.454	.670	.840	.938	.983	.995	.999						
90	.275	.493	.714	.877	.959	.990	.998	.999						
100	.294	.524	.756	.904	.973	.994	.999							

Table 3 represents the values of effect strength for sensitivity (ESS) corresponding to tabled values of Cohen's *d*, and was computed in the same manner as the parameters in the previous tables. ESS is a standardized index of classification performance on which 0 represents classification accuracy expected for a given problem by chance, and 1 represents perfect, errorless classification.[1] Table 3 enables an investigator to evaluate statistical power in experimental designs, using the more appropriate and intuitive non-parametric ESS index of effect strength.

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Table 3

Median ESS of Optimal Discrimination with One Normal Attribute and Two Classes, Obtained from Monte Carlo Analysis, Corresponding to Cohen's *d* Values: One-Tailed UniODA

n_1	Cohen's *d*													
	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3	1.4	1.5
5	.400	.400	.400	.400	.400	.600	.600	.600	.600	.600	.600	.600	.800	.800
6	.333	.333	.333	.500	.500	.500	.500	.500	.500	.667	.667	.667	.667	.667
7	.286	.286	.429	.429	.429	.429	.571	.571	.571	.571	.571	.714	.714	.714
8	.250	.375	.375	.375	.375	.500	.500	.500	.500	.625	.625	.625	.625	.750
9	.333	.333	.333	.333	.444	.444	.444	.556	.556	.556	.556	.667	.667	.667
10	.300	.300	.300	.400	.400	.400	.500	.500	.500	.600	.600	.600	.700	.700
11	.273	.273	.364	.364	.364	.455	.455	.545	.545	.545	.545	.636	.636	.636
12	.250	.333	.333	.333	.417	.417	.500	.500	.500	.583	.583	.583	.667	.667
13	.231	.308	.308	.385	.385	.462	.462	.462	.538	.538	.615	.615	.615	.692
14	.286	.286	.286	.357	.357	.429	.429	.500	.500	.571	.571	.571	.643	.643
15	.267	.267	.333	.333	.400	.400	.467	.467	.533	.533	.600	.600	.600	.667
16	.250	.250	.312	.312	.375	.437	.437	.500	.500	.562	.562	.625	.625	.625
17	.235	.294	.294	.353	.353	.412	.412	.471	.529	.529	.588	.588	.647	.647
18	.222	.278	.278	.333	.389	.389	.444	.444	.500	.556	.556	.611	.611	.667
19	.211	.263	.316	.316	.368	.421	.421	.474	.474	.526	.579	.579	.632	.632
20	.250	.250	.300	.350	.350	.400	.450	.450	.500	.500	.550	.600	.600	.650
21	.238	.238	.286	.333	.333	.381	.429	.476	.476	.524	.571	.571	.619	.619
22	.227	.273	.273	.318	.364	.409	.409	.455	.500	.500	.545	.590	.590	.636
23	.217	.261	.304	.304	.348	.391	.435	.435	.478	.522	.565	.565	.609	.652
24	.208	.250	.292	.333	.333	.375	.417	.458	.500	.500	.542	.583	.583	.625
25	.200	.240	.280	.320	.360	.400	.400	.440	.480	.520	.560	.560	.600	.640
30	.200	.233	.267	.300	.333	.367	.400	.433	.467	.500	.533	.567	.600	.633
40	.175	.225	.250	.275	.325	.350	.400	.425	.450	.500	.525	.550	.575	.600
50	.180	.200	.240	.280	.320	.340	.380	.420	.440	.480	.520	.540	.580	.600
60	.167	.200	.233	.267	.300	.333	.367	.400	.433	.467	.500	.533	.567	.600
70	.157	.186	.229	.257	.300	.329	.371	.400	.443	.471	.500	.529	.557	.586
80	.150	.187	.225	.262	.287	.325	.362	.400	.437	.462	.500	.525	.562	.587
90	.144	.178	.222	.256	.289	.322	.356	.400	.433	.467	.500	.522	.556	.589
100	.140	.180	.210	.250	.290	.320	.360	.390	.430	.460	.490	.520	.550	.580

By rule-of-thumb[4] a weak ESS ("weak effect") is defined as ESS ≤ 0.25; moderate ESS as 0.25 ≤ ESS < 0.50; strong ESS as 0.50 ≤ ESS < 0.75; and very strong ESS as ESS > 0.75. Colors are used to indicate the effect strength represented by the ESS values in Table 3: blue indicates weak effects; green indicates moderate effects; red indicates strong effects; and purple indicates very strong effects. The "saw-tooth" behavior of the power values with increasing n_1 is due to the discrete nature of both the sample and ODA.

Table 3 may be used to obtain Cohen's *d* on the basis of expected ESS. Imagine an investigator wishes to evaluate statistical power

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for a confirmatory (one-tailed) problem involving a single binary class variable, a single ordered attribute, and $n_1 = n_2 = 100$. For this problem, imagine an effect of moderate strength is anticipated. Since a moderate-strength ESS is defined as $0.25 \leq \text{ESS} < 0.50$, the lower end of this domain indicates Cohen's $d = 0.5$, and the upper end of the domain indicates Cohen's $d = 1.3$.

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